Quintessence, scalar-tensor theories and non-Newtonian gravity

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Abstract

We discuss some of the issues which we encounter when we try to invoke the scalar-tensor theories of gravitation as a theoretical basis of quintessence. One of the advantages of appealing to these theories is that they allow us to implement the scenario of a "decaying cosmological constant," which offers a reasonable understanding of why the observed upper bound of the cosmological constant is smaller than the theoretically natural value by as much as 120 orders of magnitude. In this context, the scalar field can be a candidate of quintessence in a broader sense. We find, however, a serious drawback in the prototype Brans-Dicke model with Λ added; a static universe in the physical conformal frame which is chosen to have constant particle masses. We propose a remedy by modifying the matter coupling of the scalar field taking advantage of scale invariance and its breakdown through quantum anomaly. By combining this with a conjecture on another cosmological constant problem coming from the vacuum energy of matter fields, we expect a possible link between quintessence and non-Newtonian gravity featuring violation of Weak Equivalence Principle and intermediate force range, likely within the experimental constraints. A new prediction is also offered on the time-variability of the gravitational constant.

1 Introduction

The role of the gravitational scalar field, now widely called quintessence, has been a subject of extensive studies [1,2] to understand a small cosmological "constant," as suggested by recent observations indicating an accelerating universe [3]. Much of the interest seems to be directed to the question how the dynamics of the scalar field can simulate observed behaviors. Some of the theoretical models, notably the scalar-tensor theories, have been discussed, on the other hand, as a possible solution of the "cosmological constant problem [4]." In this paper we discuss some of the crucial aspects of the arguments which we may encounter in this approach.

It seems useful to recognize that a cosmological constant may have two different origins.

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First, in almost any of the theoretical models of unification, we have no way to avoid to have Λ in the Lagrangian, with the magnitude typically of the order unity in the reduced Planckian unit system, with $c = \hbar = M_{\mathrm{p}}^{-2} = 8\pi G = 1$. This constant is too large compared with the observed upper bound, given basically by the critical density, by as much as 120 orders of magnitude. Secondly, the vacuum energy of matter fields, in the sense of relativistic quantum field theory, plays the same role as the cosmological constant. This contribution is also too large by somewhere around 60 orders of magnitude. We call them conveniently the primordial cosmological constant and the vacuum energy, respectively.

We focus upon the scalar-tensor theories of gravitation which have been discussed from many points of view, sometimes quite different from the original motivation. One of the reasons of our interest comes from the fact that it provides us with an exponential potential of the scalar field rather naturally as a direct consequence of introducing a primordial cosmological constant [4]. It offers a way to put the discrepancy of 120 orders of magnitude under control without appealing to an extreme fine-tuning of parameters. According to the "scenario of a decaying cosmological constant," today's Λ is small only because our universe is old. The number comes simply from $t^{-2} \sim 10^{-120}$ for the present age of the universe $t \sim 10\mathrm{Gy} \sim 10^{60}$ in the reduced Planckian unit system.

The effective cosmological constant, the energy of the scalar field, falls off, however, in the same way as the ordinary matter density. For this reason this model does not result in the extra acceleration of the universe. Further development is called for to understand the lower bound as well [1,2,5]. The extension based on the exponential potential still seems promising because it will inherit the decaying nature of the cosmological constant. As another potential advantage, we expect to shed a new light on the coincidence problem.

The same "success" does not seem promising, however, for the vacuum energy contribution. Moreover, we find that the prototype Brans-Dicke (BD) model suffers from a serious drawback once Λ is introduced [6]. As a remedy we proposed a modification in the scalar-matter coupling taking a risk of violating Weak Equivalence Principle (WEP). We foresee the scalar field to show up as non-Newtonian gravity, or the fifth force, once we find a way to solve the vacuum energy problem by a still unknown mechanism.

In Section 2 we sketch the solution of the prototype BD model with the primordial cosmological constant added. After explaining how the model suffers from the drawback, we outline an alternative, the dilaton model in Section 3, featuring WEP violation. Section 4 discusses non-Newtonian gravity, which is expected to arise as a residual effect. As one of key arguments, we emphasize that the force-range of the quantized field that mediates a force between local matter objects can be different from the counterpart of the classical field which governs the evolution of the universe. In the final Section 5 we discuss a modification by allowing the

self-coupling of the scalar field, as well as the predicted time variability of the gravitational constant.

2 Prototype BD model with Λ added

Consider the Lagragian for the prototype BD model with Λ added [6]:

$$\mathcal{L}_{\mathrm{BD}} = \sqrt{-g}\left(\frac{1}{2}\xi\phi^2 R - \epsilon\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \Lambda + L_{\mathrm{matter}}\right),\tag{1}$$

where $\xi > 0$ is a dimensionless constant related to the original symbol ω by $4\xi\omega = \epsilon = \mathrm{Sign}(\omega)$. Also our ϕ is a canonical field related to BD's original field φ by $\varphi = (\xi/2)\phi^2$.

It is useful to apply a conformal transformation

$$g_{\mu\nu} \rightarrow g_{*\mu\nu} = \Omega^2(x)g_{\mu\nu}, \quad \text{with} \quad \Omega = \xi^{1/2}\phi.\tag{2}$$

We have thus moved from the original conformal frame (CF) called J frame after Jordan to a new conformal frame (called E frame after Einstein) in which (1) is re-expressed as

$$\mathcal{L}_{\mathrm{BD}} = \sqrt{-g_*}\left(\frac{1}{2}R_* - \frac{1}{2}g_*^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - V(\sigma) + L_{*\mathrm{matter}}\right),\tag{3}$$

where the new canonical scalar field σ is defined by

$$\phi = \xi^{-1/2}e^{\zeta\sigma}, \quad \text{with} \quad \zeta^{-2} = 6 + \epsilon\xi^{-1},\tag{4}$$

under the condition

$$\epsilon\xi^{-1} > -6.\tag{5}$$

Also notice that the cosmological constant in J frame acts now as a potential of the scalar field;

$$V(\sigma) = \Lambda\Omega^{-4} = \Lambda e^{-4\zeta\sigma}.\tag{6}$$

In the spatially flat Robertson-Walker universe the cosmological equations in E frame are

$$3H_* = \rho_\sigma + \rho_{*m},\tag{7}$$

$$\ddot{\sigma} + 3H_*\dot{\sigma} - 4\zeta V = \eta_d\zeta\rho_{*m},\tag{8}$$

$$\dot{\rho}_{*m} + (4 - \eta_d)H_*\rho_{*m} = -\eta_d\zeta\dot{\sigma}\rho_{*m},\tag{9}$$

where we have assumed the spatially uniform σ with $\rho_\sigma = (1/2)\dot{\sigma}^2 + V$, and $\eta_d = 0, 1$ for the radiation- and dust-dominated universe, respectively. Hereafter we often attach the symbol $*$ to signify quantities in E frame.

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Notice that, unlike in J frame, we have the non-vanishing right-hand side of (9) in the dust-dominated universe. This corresponds to the geodesic equation for a point particle acquiring a nonzero right-hand side. This does not imply WEP violation because the extra force is proportional precisely to the mass.

We find an analytic solution [6]

$$a_*(t_*) = t_*^{1/2}, \tag{10}$$

$$\sigma(t_*) = \bar{\sigma} + \zeta^{-1}\frac{1}{2}\ln t_*, \tag{11}$$

$$\rho_{*m}(t_*) = \left(1 - \frac{1}{4}\zeta^{-2}\right)t_*^{-2} \times \begin{cases} \frac{3}{4}, \\ 1, \end{cases} \tag{12}$$

$$\rho_\sigma(t_*) = t_*^{-2} \times \begin{cases} \frac{3}{16}\zeta^{-2}, \\ \frac{1}{4}\left(-1 + \zeta^{-2}\right), \end{cases} \tag{13}$$

where $\bar{\sigma}$ is defined by

$$\Lambda e^{-4\zeta\bar{\sigma}} = \begin{cases} \frac{1}{16}\zeta^{-2}, \\ \frac{1}{8}\left(\zeta^{-2} - 2\right). \end{cases} \tag{14}$$

The upper and lower lines in (12)-(14) are for the radiation- and dust-dominated universe, respectively. We point out that (10)-(14) represent an attractor solution realized asymptotically. According to (13), the effective cosmological constant $\Lambda_{\text{eff}} = \rho_\sigma$ falls off like $\sim t_*^{-2}$, implementing the scenario of a decaying cosmological constant.

We find many differences from the solution obtained without Λ [7]. The scalar field continues to increase because the potential (6) keeps driving σ toward infinity, whereas σ comes eventually to rest in the radiation-dominated universe if $\Lambda = 0$. Also the presence of ρ_σ allows ρ_{*m} to be positive only for $\zeta^2 > 1/4$, from which follows $\epsilon = -1$, but still with a positive energy for the "diagonalized" σ. Rather unexpectedly, the scale factor grows in the same way both in the radiation- and dust-dominated universe.

The above condition $\zeta^2 > 1/4$ from (12) is in contradiction with the widely accepted constraint $4\zeta^2 \lesssim 10^{-3}$, or often expressed as $\omega \gtrsim 10^3$, obtained from the solar-system experiments. The constraint might be avoided if the scalar field acquires a nonzero mass giving a force-range shorter than the size of the solar system.

BD chose ϕ to be absent in L_m in J frame to ensure WEP to hold [7]. In E frame, however, mass of any particle depends on σ;

$$m_*(\sigma) = m_0\Omega^{-1} = m_0 e^{-\zeta\sigma}, \tag{15}$$

where m_0 is a constant mass in J frame. By substituting from (11), we find m_* to vary like $\sim t_*^{-1/2}$. Corresponding to the situation in which we analyze primordial nucleosynthesis, for

example, based on quantum mechanics with particle masses taken as constant, we should select J frame as a physical CF instead of E frame. The exessive time dependence may be found also in a longer time span; today's quark mass $m_{q*p} \approx 5\text{MeV} \sim 2 \times 10^{-21}$, for example, is extrapolated back to $t_* = 1$ giving as large a value as 10^9.

The behavior of the scale factor $a(t)$ in J frame can be obtained most easily by using the relations

$$dt_* = \Omega dt, \quad \text{and} \quad a_* = \Omega a. \tag{16}$$

We find $a(t) = \text{const}$. The universe looks *static* in *both* of the radiation- and dust-dominated universe. The same result can also be obtained directly in J frame.

It might be useful to discuss what the underlying reason of this result is at least for the radiation-dominated universe. First the result (10) is a direct consequence of (9) in which the right-hand side vanishes because σ has no source in the radiation-dominated universe. Secondly, (7) implies that the potential $V(\sigma) \sim \Omega^{-4}$ must behave like t_*^{-2}, from which follows $m_* \sim \Omega^{-1} \sim t*^{-1/2}$. We now find that $a_*(t_*)$ and m_*^{-1} grows in the same way. The meter stick provided by m_*^{-1} expands in the same way as the universe. Using this kind of meter stick corresponds exactly to living in J frame.

This conclusion may not be final, because it depends on the simplest choice of the non-minimal coupling, as well as the assumption of no self-coupling of ϕ. It seems still devastating because the simplest choice is so remote from what we expect from the standard cosmology. We find it far from easy to understand why the universe expands in the manner of the standard model. It might be worth looking for a remedy from quite a different point of view, still on a simple theoretical basis.

3   Dilaton model

A possible alternative might be found by favoring E frame in which we have the standard result $a_*(t_*) = t_*^{1/2}$ for the radiation-dominated universe. Let us expect that the E frame mass term is given by

$$\mathcal{L}_{mq} = -m_{q\dagger}\sqrt{-g_*}\bar{q}_* q_*, \tag{17}$$

where $m_{q\dagger}$ is a *constant* mass of the quark, for example. Obviously, this can be conformally transformed back to the Yukawa interaction in J frame;

$$\mathcal{L}_{mq} = -\xi^{1/2} m_{q\dagger}\sqrt{-g}\bar{q}q\phi. \tag{18}$$

Notice that the coupling constant is dimensionless, as verified by re-installing M_{P}^{-1}. Allowing the scalar field to enter the matter Lagrangian should, however, endanger WEP, but without spoiling Equivalence Principle at the more fundamental level stating that tangential space to

curved spacetime should be Minkowskian. Also violation of WEP as a phenomenological law can be tolerated within the constraint obtained from the fifth-force-type experiments [8].

This favorable result is lost, however, once the effect of interactions among matter fields is taken into account. The QCD calculation, with the help of dimensional regularization, corresponding to the 1-loop diagram in Fig. 1 yields the linear term [6]

$$\mathcal{L}_{mq1} = \sqrt{-g_*}\,\zeta_q m_{q\dagger} \bar{q}_* q_* \sigma, \tag{19}$$

where

$$\zeta_q = \zeta \frac{5\alpha_s}{\pi} \approx 0.3\zeta, \tag{20}$$

with $\alpha_s \approx 0.2$, the QCD counterpart of the fine-structure constant. WEP violation is explicit because ζ_q depends on α_s which is specific to the quark.

The coupling strength indicated by (20) is not considerably weaker than ζ in the linear term in (15). It still seems sufficient to suppress particle masses at the earliest universe to a "reasonable" size. Even $M_{*sb} \sim 1\text{TeV} \sim 4 \times 10^{-16}$ for the mass scale of supersymmetry breaking remains $\sim 10^{-6}$ at $t_* = 1$, if (19) is justified to be exponentiated for large σ. We need, however, a detailed analysis on whether the many-loop calculation for terms higher order in σ would result in the exponential function. A different asymptotic behavior might emerge.

From a more realistic point of view, however, we may focus on the nucleon mass. By considering that the quark mass content of a nucleon is relatively small, $\sim 60\text{MeV}$, we estimate

$$\zeta_N \approx 0.02\zeta, \tag{21}$$

suggesting that E frame can be a physical CF to a good approximation, though we present a more detailed analysis later.

The small value of ζ_N/ζ, as well as ζ's for other particles with weaker interactions, might help to understand why the exponent in $a_*(t_*) \sim t_*^{\alpha_*}$ for the dust-dominated universe as given by [6]

$$\alpha_* = \frac{1}{6}\left(4 - \frac{\bar{\zeta}}{\zeta}\right) \tag{22}$$

is close to 2/3, where $\bar{\zeta}$ means an average of ζ's for particles comprising the dust matter.

The vacuum energy of matter fields is estimated roughly of the order of $E_{ve} \sim M_{*sb}^4$ According-ing to (20) we may expect $E_{ve} \sim 10^{-24}$ at the earliest epoch. Multiplying this by $e^{-4\zeta_q\sigma}$ will give another potential $V_1(\sigma)$. Due to $\zeta_q < \zeta$, however, the new potential will soon overwhelm $V(\sigma)$, eventually reproducing the excess by 60 orders of magnitude at the present epoch. The "vacuum energy problem" seems to call for a novel mechanism which is yet to be discovered.

We also add that the calculation leading to (19) and (20) is closely related to the trace anomaly [9]. In fact the J frame Lagrangian with the matter part (18) but ignoring Λ, for

the moment, is invariant under global scale transformation (dilatation). We find that σ in E frame plays the role of the associated massless Nambu-Goldstone boson (dilaton). Due to the quantum correction, the dilatation symmetry is ultimately broken explicitly, making σ the pseudo Nambu-Goldstone boson which is massive.

4 Non-Newtonian gravity

In E frame as an approximately physical CF, we expect that the scalar field σ is decomposed into the sum of the cosmological background part $\sigma_b(t_*)$ and the locally fluctuating component $\sigma_f(x)$;

$$\sigma(x) = \sigma_b(t_*) + \sigma_f(x). \tag{23}$$

Substituting this into (19) and using (15) with σ replaced by $\sigma_b(t_{*\mathrm{p}})$ to give the quark mass m_{*q}, we obtain

$$\mathcal{L}_{mqf} = \sqrt{-g_*}\,\zeta_q m_{*q}\bar{q}_*q_*\sigma_f, \tag{24}$$

which implies that σ_f mediates a force among matter objects.

We may also apply a familiar field theoretic calculation obtaining the self-mass μ_f arising from a quark loop, for example, as estimated to be

$$\mu_f^2 \sim m_{*q}^2 M_{*sb}^2. \tag{25}$$

By using today's values for the masses, we obtain $\mu_f \sim 0.84 \times 10^{-36} \sim 2.1 \times 10^{-9}\mathrm{eV}$, and the corresponding force range $\lambda = \mu_f^{-1} \sim 1.2 \times 10^{36} = 9.6 \times 10^3\mathrm{cm} \approx 100\mathrm{m}$. We should allow, however, a latitude of several orders of magnitude due to ambiguities in evaluating the self-energy. It seems nevertheless unlikely that the force-range is as larger as the size of the solar-system to allow the solar-system experiments to constrain the coupling strength ζ.

It has been argued, on the other hand, that the force mediated by σ is long-range, because the second derivative of $V(\sigma)$ is extremely small [2]. This can be derived first by noting that the left-hand side of (7) is $\sim t_{*\mathrm{p}}^{-2}$ as long as the universe at the present time $t_{*\mathrm{p}}$ expands according to a power law, placing an upper bound on V on the right-hand side. If the potential is sufficiently flat, like the exponential potential, for example, the mass squared defined by the second derivative should be of the same order of magnitude as V itself. The corresponding force-range is $\sim t_{*\mathrm{p}}$, which is the size of the whole visible universe.

The squared mass given by (25) is overwhelmingly larger than $\sim t_{*\mathrm{p}}^{-2}$. In this connection we point out, however, that the above argument for a long-range force applies to σ_b, a classical background field supposed to obey a nonlinear equation, whereas (25) is related to the solution of a linear harmonic oscillator, which is the basis of the concept of a quantum. The two kinds of mass can be entirely different from each other. A well-known example is provided by the

sine-Gordon equation in 2-dimensions [10]. The quantized field has "mesonic excitations" at each of the sinusoidal potential minima, with the mass squared given by the second derivative of the potential. Quite apart from them, there are classical soliton solutions, which connect the potential minima. The mass of each of such solutions is in fact different from the mass of the meson.

The analogy is far from complete in our model. The squared mass $\sim t_*^{-2}$ has nothing to do with the soliton mass. We even do not know if there is a soliton-type solution of our cosmological equation, although we might expect a mechanism of nonlinear dynamics particularly for the effect of the vacuum energy that includes the effect of the fully nonlinear extension of (19). What still interests us is that the soliton solution shows the behavior entirely different from the propagation of the mesonic excitation. In the same way we may anticipate the slow evolution of $\sigma_b(t_*)$ instead of the oscillatory behavior. Without entering any further details at this moment, we simply propose a conjecture that the fluctuating component acquires a nonzero mass given by (25) in a way compatible with the slow rolling of the background field.

Given the urgency of accommodating WEP violation in the prototype BD model and the inevitability of solving the vacuum-energy problem, it might be a unique consequence of a viable model of scalar-tensor theories with a cosmological constant to have σ_f showing itself quintessentially as non-Newtonian gravity, or the fifth force in its scalar version [8,11].

The phenomenological analyses can be made most conveniently in terms of the static potential between two nuclei a and b;

$$V_{5ab}(r) = -\frac{Gm_a m_b}{r}\left(1 + \alpha_{5ab}e^{-r/\lambda}\right),$$ (26)

where the coefficient α_{5ab} is given basically by the one between two nucleons;

$$\alpha_{5N} = 2\zeta_N^2.$$ (27)

According to (21) we have $\alpha_{5N} \sim 10^{-3}$ for $\zeta = 1$, which seems already on the verge of an immediate exclusion [8], though the conclusion might be premature in view of uncertainties in the force-range as well as estimating composition-dependence coming from the nuclear binding energies. It seems nevertheless interesting to suggest a possible link between the cosmological constant problem and non-Newtonian gravity.

5 Discussions

The Λ term in (1) may depend on ϕ, as suggested by some examples of higher-dimensional theories. In other words, we may allow self-interaction of the scalar field. Let us consider a monomial $\phi^\ell\Lambda$, for simplicity. After the conformal transformation (2) we obtain

$$V(\sigma) = \Lambda\xi^{-\ell/2}e^{-4\zeta'\sigma},$$ (28)

where

$$\zeta' = \left(1 - \frac{\ell}{4}\right)\zeta. \tag{29}$$

Except for the choice $\ell = 4$, we have the same exponential potential only with a different coefficient ζ'. The relation (4) remains the same as before. The solution (10)-(14) are still correct if we replace ζ everywhere by ζ' for the radiation-dominated universe, while we encounter some complications for the dust-dominated universe, because ζ on the right-hand sides of (8) and (9) remain unchanged. Likewise, the constant ζ that determines the matter coupling in (15) and (20) is still ζ. As a consequence, we now have $m_* \sim t_*^{-(\zeta/\zeta')/2}$.

In the previous discussions, the strength of the matter coupling is constrained from below because $|\zeta| > 1/2$ from the physical condition in (12). We can relax this constraint by noting that the condition is now for ζ' and (29) allows $|\zeta| < |\zeta'|$ for $\ell < 0$ or $\ell > 8$. This might make it easier for α_{5N} as given by (27) to be consistent with experiments. We may have even more flexibility if we allow a more general function of ϕ multiplied by Λ in (1).

For the prototype model, we showed that the universe is static in J frame. This conclusion is subject to a change for a nonzero ℓ. We find $a(t) = t^\alpha$ with $\alpha = ((1/2), \text{or } (2/3)) \times (\ell/(\ell - 2))$ if $\ell \neq 2$, for the radiation- or dust-dominated universe, respectively, if $\ell \neq 2$, while $a(t)$ shows an exponential behavior if $\ell = 2$ [6]. In this way departing from WEP may not appear so much urgent. However, obtaining α which agrees with the standard value within the difference of $\pm 10\%$, for example, requires either $\ell < -18$ or $\ell > 22$, somewhat extreme choices. Also α is negative for $0 < \ell < 2$.

In Section 3, we were content with having E frame as an *approximately* physical CF. Strictly speaking, however, we should move to another CF in which particle masses stay constant. Let us do this by assuming again the exponential dependence for $\ell = 0$;

$$m_{*N}(t_*) \sim t_*^{-\delta/2}, \tag{30}$$

where $\delta = \zeta_N/\zeta$. Remember that without WEP masses of different species of particles may behave differently. The time variable $d\tilde{t}$ measured in units of m_{*N}^{-1} is defined, analogously to (16), by $d\tilde{t} = dt_*/m_{*N}^{-1}$, yielding

$$\tilde{t} \sim t_*^{1+\delta/2}. \tag{31}$$

By using also the second relation in (16) we obtain

$$\tilde{a} \sim m_{*N}a_* \sim \tilde{t}^{\tilde{\alpha}}, \tag{32}$$

where the exponent

$$\tilde{\alpha} \approx \frac{2}{3}\left(1 - \frac{3}{2}\delta\right), \tag{33}$$

has been derived from (22) for the dust-dominated universe in E frame, keeping only terms proportional to $\delta \approx 0.02$ shown in (21).

A small deviation from 2/3 as in (33) may not be seriously important. We then try to show that the gravitational constant is now time-dependent to the extent that it is close to the observational upper bounds available so far. For this purpose we consider (19) assumed to be exponentiated with q replaced by N. Under the conformal transformation $g_{*\mu\nu} \rightarrow \tilde{\Omega}^2 \tilde{g}_{\mu\nu}$, the mass is transformed as $m_* \rightarrow \tilde{m} = m_* \tilde{\Omega}^{-1}$. We therefore choose $\tilde{\Omega} \sim m_{*N}$. Ignoring terms of higher order in δ, we obtain

$$\frac{1}{2}\sqrt{-g_*}R_* \approx \frac{1}{2}\sqrt{-\tilde{g}}\tilde{\Omega}^{-2}\tilde{R}, \tag{34}$$

in which we identify the gravitational constant in the tildered CF as

$$8\pi\tilde{G} = \tilde{\Omega}^2 \sim t_*^{-\delta}. \tag{35}$$

Further using $\tilde{t} \sim t_*$ obtained from (31) by omitting the δ-dependence, we finally find

$$\frac{\dot{\tilde{G}}}{\tilde{G}} \sim -\delta\tilde{t}^{-1}, \tag{36}$$

which can be compared with the observed upper bound $(0.2 \pm 0.4) \times 10^{-11}\text{y}^{-1}$ [12]. Further improving the accuracy will test the proposed theoretical model.

Note that we have applied the conformal transformation only in the context of the classical background field, leaving (24) still accepted as the coupling of the quantized field almost unaffected.

We add, however, that there is a theoretical model [5] featuring the scalar field that stays nearly constant for some time duration, allowing us to understand an extra acceleration of the universe, as indicated by recent observations. During this period supposed to cover the present epoch, we can avoid the issue of choosing a CF, also predicting the time variation of the gravitational constant at the level much lower than 10^{-10}y^{-1}. We reasonably expect again that the matter coupling of σ_f remaining nearly the same even for the modified version of the model.

We point out, on the other hand, that it might take some time for the observational studies before the presence of the cosmological constant will be finally established. We must still answer the question how the discrepancy of 120 orders of magnitude is avoided. The issue of $\dot{G}/G$ is related to this part of the question.

Acknowledgments

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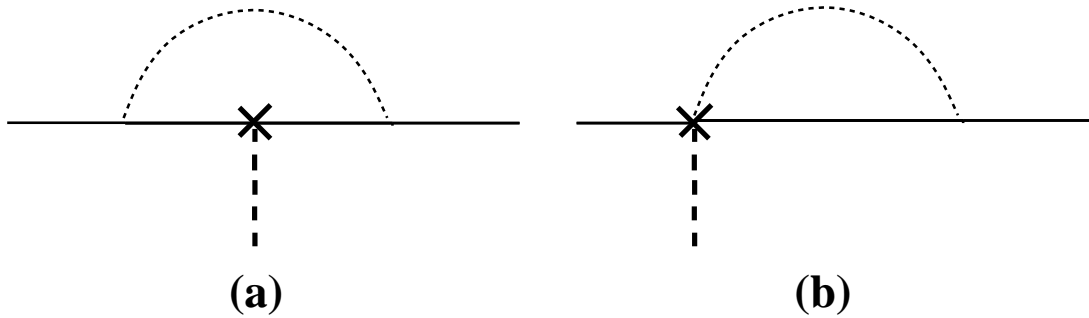[12] R.W. Hellings, *et al.*, Phys. Rev. Lett. **51**, 1609(1983).

(a) **(b)**

Figure 1: One-loop diagrams contributing to the anomaly, in the framework of N-dimensional regularization. We started with spacetime dimensionality $N \neq 4$, which renders loop integrals expressible in temrs of Gamma functions $\Gamma(2 - N/2)$. On the other hand, vertices denoted by crosses, the mass term in (a) while the interaction term in (b), are proportional to $N - 4$, which multiplies with the pole in the gamma function to produce a finite result [6]. The solid and dotted lines represent the quark and the gluon field, respectively, while the dashed line is for σ.